TRANSMATH®

Targeted, Multisensory Math Intervention
Curriculum for Students in Grades 5-10

What the IES Guide & Research Tell Us

VOYAGER SOPRIS LEARNING®

What is the IES Guide?

In 2021, the Institute of Education Sciences Practice Guide (or IES Guide), *Assisting Students with Mathematics: Interventions in the Elementary Grades*, was released by What Works Clearinghouse™. It provides recommendations for educators about addressing classroom challenges based on research, experiences of practitioners, and the expert opinion of a panel of nationally recognized experts. Each guide supports educators by providing a summary of existing research, a discussion of best practices supported by research evidence, and key examples.

THE RECOMMENDATIONS INCLUDE:

Recommendation 1: Systematic Instruction: Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Recommendation 2: Mathematical Language: Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

Recommendation 3: Representations: Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Recommendation 4: Number Lines: Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

Recommendation 5: Word Problems: Provide deliberate instruction about word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

Recommendation 6: Timed Activities: Regularly include timed activities as one way to build fluency in mathematics.

The Relationship between *TransMath*® and the *2021 IES Guide: Assisting Students with Mathematics: Interventions in the Elementary Grades*

It should be noted that the 2021 guide is a revision of the 2009 guide, which was originally written for K–8 audience of students. The panel decided to limit the current guide to K–6, which had most empirical support. However, all recommendations are valid for middle grades.

MATH EDUCATION EXPERT

Dr. John Woodward is a nationally recognized mathematics author, writer, and speaker. He is the past dean of the school of education and professor emeritus at the University of Puget Sound in Tacoma, WA.

As a researcher, Dr. Woodward focused on mathematics interventions for academically low-achieving students, particularly in elementary and middle grades. He is the senior author of *TransMath*, a math intervention program for middle school students. He also is the co-developer of *NUMBERS*, a math professional development program for K–8 teachers.

Practical Support for Educators

The goal of the guide is to support educators in implementing effective strategies and interventions in their classrooms and schools by:

1. **Providing a series of instructional recommendations that are supported by high-quality educational research**
2. **Translate recommendations into a usable form that can be understood by educators**

Educators often teach a range of academic subjects as well as develop IEPs, manage behavior, and a host of other duties. Most importantly, *educators are not curriculum writers*. Therefore, it is critical that if educators are to help struggling students achieve today's standards, they need detailed help beyond the practice guides.

While these recommendations are directed toward the elementary grades, the topics still apply to middle school mathematics. This is particularly true of students who struggle in math and are receiving special education services. For example, it is common that students in the middle grades are being taught topics like fractions—an elementary topic in the standards—in intervention settings. Furthermore, other middle-grade topics like integers involve the use of number lines. Ratios and proportions as well as algebra involve a structured approach to word-problem solving. The use of representations applies to mathematical topics across all grade levels. Finally, struggling students still need timed activities in their math facts. For these reasons, the recommendations in this guide extend to the intended grade levels in *Transmath*.

With that in mind, we've put together this 'road map' to help you see how *TransMath* supports teachers' efforts to move students across key, standards-based topics in the elementary and middle grades.

What is *TransMath*?

TransMath Third Edition is a comprehensive math intervention curriculum that targets middle and high school students who lack the foundational skills necessary for entry into algebra and/or who are two or more years below grade level in math. Using a dual-topic approach, *TransMath* provides a balance of conceptual learning of number concepts and problem-solving applications which accelerate students to more advanced math, from number sense to rational numbers, to understanding algebra. Students who struggle with math will find new concept mastery and confidence with *TransMath*.

Having a program like TransMath that breaks [math] down is amazing...When my students say, 'I can't do fractions,' and then by the end of the lesson they're getting 95 percent and saying, 'Yes, I can,' it's really great to see."

—Sarah, Middle School Teacher, New Mexico

THE *TRANSMATH* APPROACH

- ✔ Explicit instruction and multisensory strategies deepen conceptual understanding and build problem-solving proficiency
- ✔ Math lessons and models are embedded to support teacher preparation and strengthen teacher content knowledge
- ✔ Whole-class, interactive-learning is facilitated by providing access to digital tools to increase opportunities for mathematical discourse and peer learning
- ✔ Students and teachers have eBook access to support their learning while also providing greater interaction

TRANSMATH BRIDGES THE GAP FOR MIDDLE AND HIGH SCHOOL STUDENTS

TransMath benefits students in grades 5–10 who require immediate support in building the foundational skills necessary for successful entry into algebra. This includes any student scoring two or more years below grade level on state standardized tests. *TransMath* also supports teachers with strategies, such as differentiation, and structured lessons to help meet the needs of these students.

EVIDENCE BASED

TransMath has research-proven, effective elements that accelerate students toward grade-level mathematics through strong teacher support with lesson-by-lesson models. The instructional principles that form the foundation of *TransMath's* pedagogy are supported by research and include:

- Concrete and Visual Representations
- The Controlling of Cognitive Load
- Distributed Practice
- Varied Opportunities for Communication
- Multiple Forms of Assessment

How does *TransMath* align to the IES Guide's recommendations and what steps can be taken to achieve them?

RECOMMENDATION 1: SYSTEMATIC INSTRUCTION

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

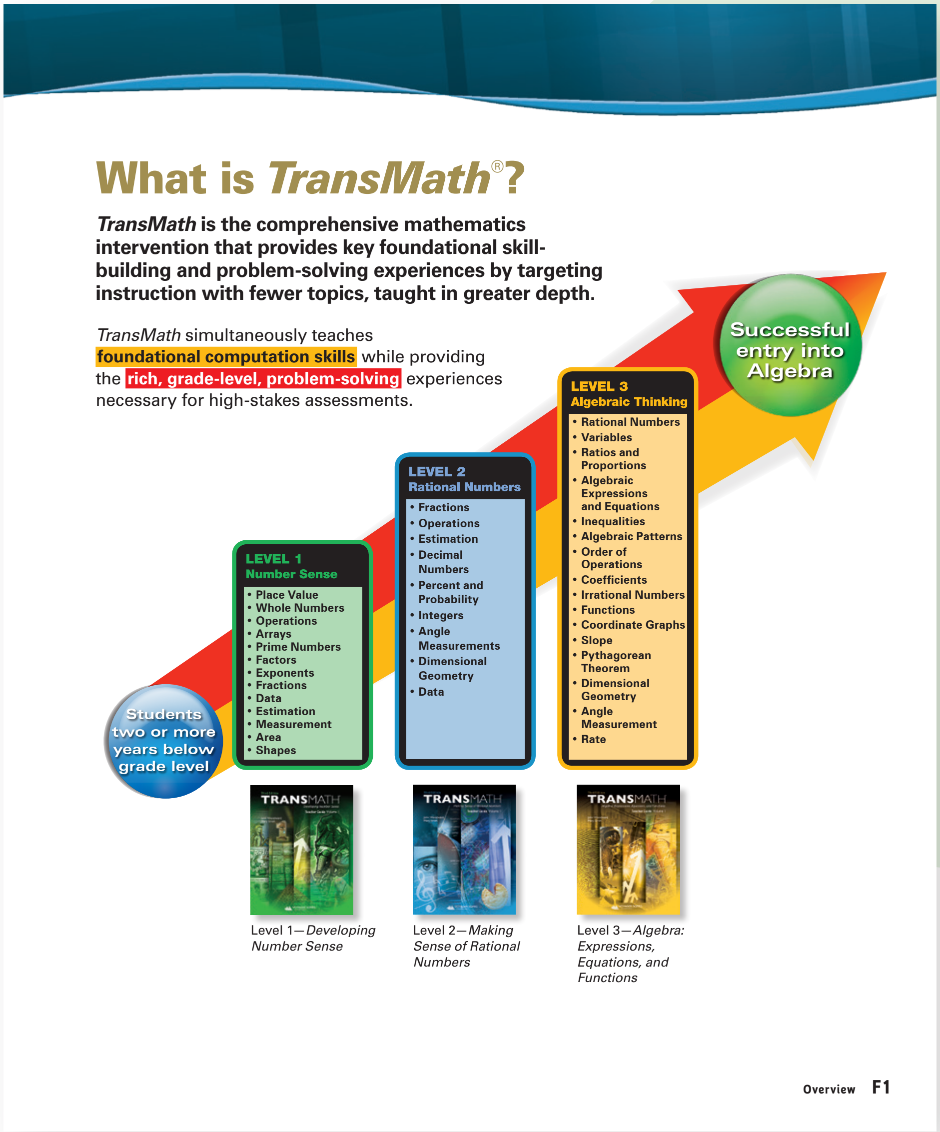
Unfortunately, many interventions for struggling students in the middle grades tend to focus on math facts drills and highly procedural practice on whole number operations. The almost exclusive focus on these skills tends to come at the expense of progress in other topics articulated in state and national standards.

In contrast, *TransMath* was designed and revised around the Common Core State Standards for Mathematics (CCSSM). Among other things, that means more topics are taught at a conceptual level and more problem solving is included. One of the strengths of *TransMath* is that it gives teachers what they need to move from operations on whole numbers up through other topics in numbers (fractions, integers, early algebra), geometry, basic statistics, and ratios and proportions. This systematically designed direction gives teachers the structured support they need to teach these topics, particularly at a conceptual level, and implement what is recommended in the guide.

Examples

Designed specifically for systematic instruction, *TransMath* clearly relates to Recommendation 1 by "review and integrate previously learned content throughout the intervention to ensure that students maintain understanding of concepts and procedures." Each lesson builds on the other in a way that supports distributed practice on concepts and procedures. Even the homework has cumulative review as a daily activity.

The "two topics" design of each chapter of *TransMath* complies with the way the recommendation encourages "sequence(d) instruction so that the mathematics students are learning builds incrementally."



What is *TransMath*®?

TransMath **is the comprehensive mathematics intervention that provides key foundational skill-building and problem-solving experiences by targeting instruction with fewer topics, taught in greater depth.**

TransMath simultaneously teaches **foundational computation skills** while providing the **rich, grade-level, problem-solving** experiences necessary for high-stakes assessments.

Level 1—*Developing Number Sense*

Level 2—*Making Sense of Rational Numbers*

Level 3—*Algebra: Expressions, Equations, and Functions*

Overview F1

This image from the Teacher's Guide (F1 in Level 2) illustrates the systematic instruction per Recommendation 1 guidelines.

Lesson 12

Problem Solving: ▶ When Remainders Are Important

How do we know when to consider the remainder in our answer when solving a word problem?

(*Student Text*, pages 264–266)

Connect to Prior Knowledge

Ask students to define a remainder and give an example of it. It's the amount left over when the divisor does not evenly divide the dividend. Have students write problems with remainders on the board and solve them. Remind students to think about near facts when solving these problems. How do we write the remainder? We have been writing remainders with an **R** in front, such as **R2** for a remainder of 2.

Link to Today's Concept

Tell students that today they will look at remainders in word problems and determine if the remainders can be ignored or if they need to be considered. Sometimes, the leftover part is not important. Other times, it's very important.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate why the context of a word problem is important in the following way:

- Have students look at the multiple-choice problem at the top of page 264 of the *Student Text*. Read through the problem together carefully. Be sure students understand the problem and what it is asking. Explain that most students taking this test got the answer wrong even though they divided correctly. Why? They did not consider what to do with the remainder in a word problem.

Lesson 12 | Problem Solving: ▶ When Remainders Are Important

▶ Problem Solving: When Remainders Are Important

How do we know when to consider the remainder in our answer when solving a word problem?

Several years ago, 45,000 eighth-grade students were given a problem like this:

Problem:

A hotel provides free bus service to the airport for the guests who stay at the hotel. One day, 248 guests need a ride to the airport. Each bus can hold 20 people. How many buses are needed to take everyone to the airport?

Select the correct answer:

(a) 12 buses
(b) 12 R8 buses
(c) 13 buses
(d) 20 buses

It is amazing, but 7 out of 10 students chose **b**. That means they did the long division correctly for this problem.

$$\begin{array}{r} 12 \text{ R8} \\ 20\overline{)248} \\ -20 \\ \hline 48 \\ -40 \\ \hline 8 \end{array}$$

BUT THESE STUDENTS ALL GOT THE WRONG ANSWER! Why? They did the math correctly.

Think about the problem for a moment. The problem states that 248 people need to be driven from the hotel to the airport. *All of the people* need to be taken to the airport. A "remainder 8 bus" does not exist. Also, a surprising number students chose **a**. That means 12 buses are used.

$$\begin{array}{r} 20 \\ \times 12 \\ \hline 40 \\ +200 \\ \hline 240 \end{array}$$

264 Unit 4 • Lesson 12

- Tell students that we have to think about the context when we solve real-world division problems with remainders. In this case, when pe... airp... the... rem... bus... to c... the...

This example describes how students should think about the concept of a remainder at a higher level. The text in the Teacher's Guide and student text presents a detailed guide for thinking about the issue.

Demonstrate

- Have students look at **Example 1** on page 265 of the *Student Text*. In this example, we talk about a context where the remainder does not matter. Read through the problem carefully together with students. Be sure they see why we do not need to consider the remainder. The answer is 5 R42. There are 1,000 meters in a kilometer–42 meters is not that important. It is insignificant so we ignore the remainder.
- Have students look at the answer as shown by the number line to reinforce how unimportant the remainder is in the context of this word problem.

Lesson 12

That means 240 of the 248 people are taken to the airport. The last 8 people are left at the hotel. All of the people were not taken to the airport.

The correct answer is **c**. The hotel needs 13 buses. There are 20 people on 12 of the buses, and the last bus only has 8 people. That way everyone gets from the hotel to the airport.

There are times when we don't worry about the remainder, and there are times when we have to think about what we do with the remainder.

Example 1

In what situations do remainders not matter?

Big track meets around the world measure the distance people run in meters and kilometers. There is the 100-meter dash for short races. Long-distance races are measured in kilometers.

1,000 meters = 1 kilometer

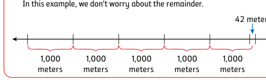
LaShawn is a professional runner, and he is practicing for a long-distance race that will be held in Germany next summer. Today he used a fitness wristband to measure the exact distance he ran. The wrist band showed that he ran 5,042 meters. About how many kilometers did he run?

The problem shows that he ran 5,042 meters. There are 1,000 meters in each kilometer (or group), so how kilometers (or groups) did he run?

$$\begin{array}{r} 5 \text{ R42} \\ 1{,}000\overline{)5{,}042} \\ -5{,}000 \\ \hline 42 \end{array}$$

LaShawn ran about 5 kilometers. The extra 42 meters is pretty small compared to the 1,000 meters in a kilometer.

In this example, we don't worry about the remainder.

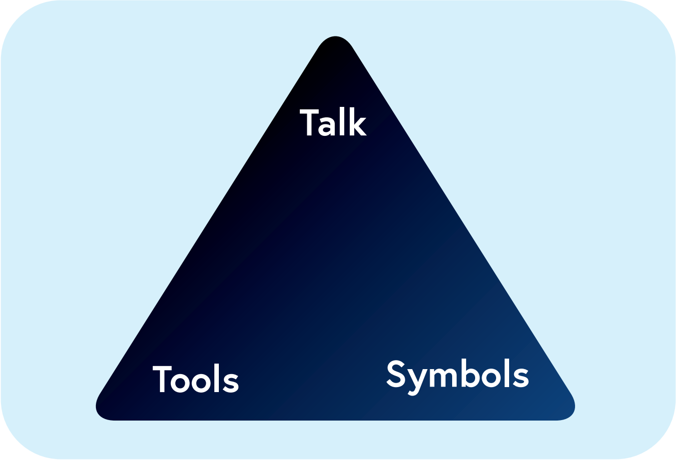


Unit 4 • Lesson 12 265

RECOMMENDATION 2: MATHEMATICAL LANGUAGE

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

In addition to systematic instruction, *TransMath* encourages the proper use of mathematical terms and concepts. The authors were selective about what terms are important for students to know, and go beyond simple "vocabulary" so that students develop their understanding of mathematics based on this image:



In other words, terms do not "stand by themselves." Instruction needs to be orchestrated so students see the direct connection between the words and concepts they are using; the tools (e.g., place value coins, Cuisenaire rods, number lines) associated with those words and concepts; and how all of the terms look pertaining to mathematical symbols or notation.

For example, it is one thing to say, "fractions are a part to whole relationship," which is technically correct, and another to show what the statement means with Cuisenaire rods and associated fraction symbols. High-quality, random assignment research on *TransMath* (Jayanthi et al., 2021) validated this connection and how it had a significant and positive effect on classroom talk with students in the experimental group versus students who were not.

Examples

Lesson **10** | Repeating Decimal Numbers

Monitoring Progress: Quiz 2

Lesson Planner

Vocabulary Development

repeating decimal numbers

Skills Maintenance

Converting Decimal Numbers to Fractions, Relationships between Lines

Building Number Concepts:

Repeating Decimal Numbers

We look at converting fractions to decimal numbers with a calculator when the resulting decimal number is a repeating decimal number. Students learn that sometimes when we convert fractions to decimal numbers with a calculator, we get many digits, and the digits can repeat in a predictable way. We call these repeating decimal numbers.

Students learn to represent these decimal numbers with a line over the top of the repeating part of the decimal number. Students need to know and understand repeating decimal numbers and learn ways to work with them efficiently.

Objective

Students will analyze repeating decimal numbers.

Monitoring Progress:

Quiz 2

Distribute the quiz, and remind students that the questions involve material covered over the previous lessons in the unit.

Homework

Students convert fractions to decimal numbers, convert decimal numbers to fractions, and convert fractions to decimal numbers using a calculator. In Distributed Practice, we revisit whole number operations.

Lesson 10 | Skills Maintenance

Name ______ Date ______

Skills Maintenance
Converting Decimal Numbers to Fractions

Activity 1

Convert the decimal numbers to fractions.

1. 0.1 $\frac{1}{10}$ 2. 0.05 $\frac{5}{100}$ 3. 0.100 $\frac{100}{1,000}$
4. 0.050 $\frac{50}{1,000}$ 5. 0.002 $\frac{2}{1,000}$ 6. 0.300 $\frac{300}{1,000}$

Relationships between Lines

Activity 2

Circle the correct term that describes each pair of lines.

1. Parallel / Perpendicular / Neither
2. Parallel / Perpendicular / Neither
3. Parallel / Perpendicular / Neither
4. Parallel / Perpendicular

Unit 5

Skills Ma…
Converting…
Relationsh…
(Interactive…
Activity …
Students c…
Activity …
Students id…

Lesson 13

- Use student responses to guide the discussion. The point of the discussion is to reinforce the benefits of rounding and using benchmarks when dealing with confusing numbers, such as decimal numbers with many decimal places.
- Show how to round the number to the hundredths place, 0.47. Discuss how we might talk about this number. Explain how this number relates to the benchmark 0.50. Tell students that benchmarks like 0.50 help us gain good number sense about decimal numbers.

How do benchmarks help us understand decimal numbers? *(continued)*

Demonstrate

- Have students look at the examples of identifying benchmarks for decimal numbers on page 369 of the *Student Text*. **Example 1** looks at the decimal number **0.79**. Point out the decimal number on the number line, and tell students that it is close to the benchmark 0.75. Remind students that 0.75 is equal to the common fraction $\frac{3}{4}$.
- Move to **Example 2** and **Example 3**. We compare decimal numbers using **$0.\overline{3}$** and **$0.\overline{6}$** as common benchmarks. Make sure students realize that these relate to the $\frac{1}{3}$ and $\frac{2}{3}$ fractional benchmarks.
- Make sure students realize that they can use fraction or decimal number benchmarks interchangeably (because they represent the same location on the number line). They should use the representation that makes the most sense for the context in which they are working.

Lesson 13

Example 1

Find the decimal number 0.79 on the number line and the nearest decimal benchmark.

The decimal number 0.79 is close to the benchmark 0.75, which we know is equal to the common fraction $\frac{3}{4}$.

Example 2

Find the decimal number 0.35 on the number line and the nearest decimal benchmark.

The decimal number 0.35 is close to the benchmark $0.\overline{3}$, which we know is equal to the common fraction $\frac{1}{3}$.

Example 3

Find the decimal number 0.63 on the number line and the nearest decimal benchmark.

The decimal number 0.63 is close to the benchmark $0.\overline{6}$, which we know is equal to the common fraction $\frac{2}{3}$.

We can use fraction or decimal number benchmarks when we find decimal numbers on a number line. We use whichever makes the most sense for the decimal numbers we are looking at.

Apply Skills: Turn to *Interactive Text*, page 233.

Reinforce Understanding: Use the *Unit 5 Lesson 13 Teacher Talk Tutorial* to review lesson concepts.

Unit 5 • Lesson 13 369

Check for Understanding

Engagement Strategy: Think, Think

Ask students the following questions. Tell them that one of them will be called on to answer a question. Ask them to listen for their name to be called. After each question, allow time for students to think of the answer. Then call on a student.

Ask:

What common benchmark is the decimal number 0.53 close to? *(0.50)*

What common benchmark is the decimal number 0.24 close to? *(0.25)*

What common benchmark is the decimal number 0.65 close to? *(0.75)*

Reinforce Understanding

Remind students that they can review lesson concepts by accessing the online *Unit 5 Lesson 13 Teacher Talk Tutorial.*

At a broad level, TransMath contains numerous engagement strategies for fostering classroom talk.

In this example (page 600, Teacher's Guide, Volume 2, Level 2), students use calculators to convert fractions to decimals and make sense of the results with benchmark numbers. The guide includes a "think, think" strategy and questions for checking understanding. Talk strategies help with formative assessment, encouraging teachers to "listen for" responses that reveal student comprehension.

RECOMMENDATION 3: REPRESENTATIONS

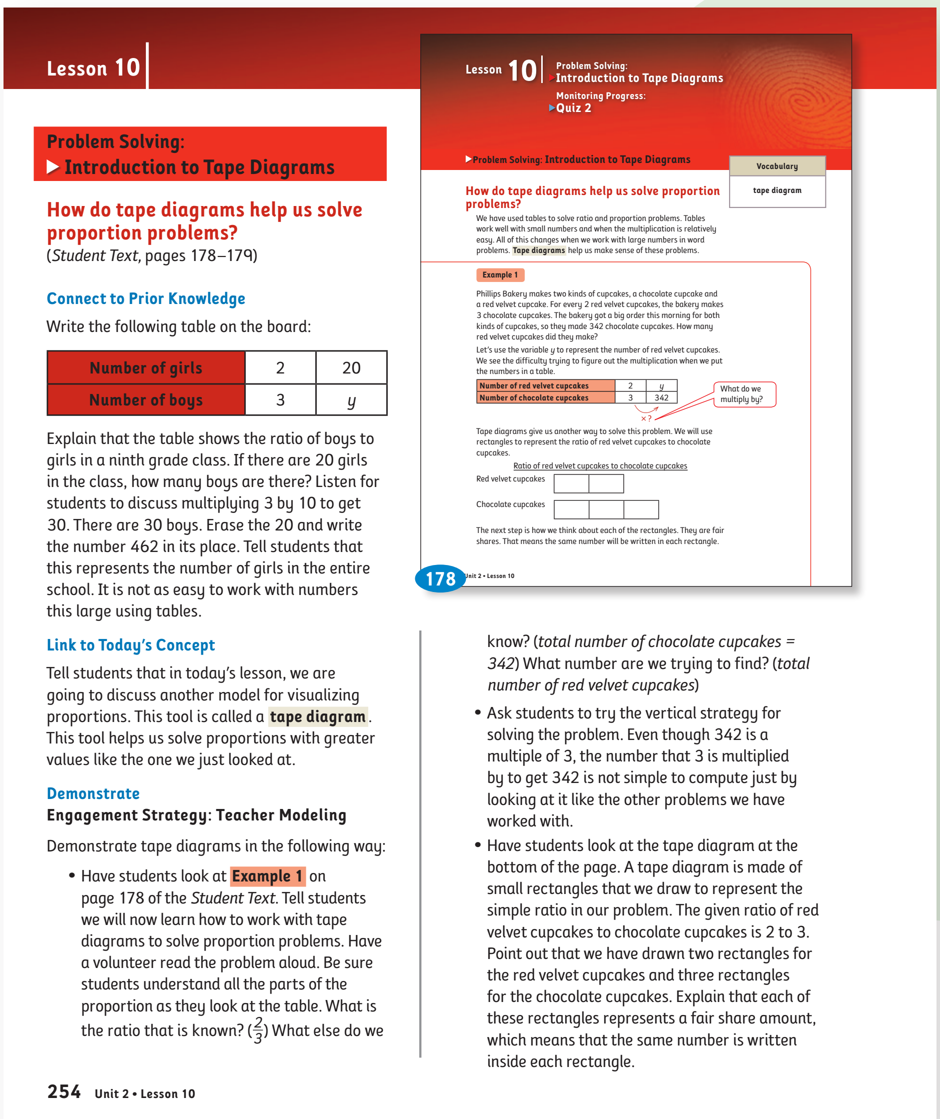
Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Throughout *TransMath*, you'll find representations that enhance student understanding. Here is a sampling of the tools as they relate to mathematical concepts:

- **Operation on Whole Numbers:** Place value coins
- **Fractions:** Cuisenaire rods, number lines
- **Integers:** Algebra tiles
- **Ratios and Proportions:** Tape diagrams, tables
- **Pre-algebra/Algebra:** Algebra tiles
- **Problem Solving:** Schematic diagrams

The use of these tools is also reinforced in tutorials, which clearly complies with the recommendation and the way it encourages teachers to "provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as 'thinking tools.'"

Examples



Lesson 10

Problem Solving:
▶ Introduction to Tape Diagrams

How do tape diagrams help us solve proportion problems?
(*Student Text*, pages 178–179)

Connect to Prior Knowledge

Write the following table on the board:

Number of girls	2	20
Number of boys	3	y

Explain that the table shows the ratio of boys to girls in a ninth grade class. If there are 20 girls in the class, how many boys are there? Listen for students to discuss multiplying 3 by 10 to get 30. There are 30 boys. Erase the 20 and write the number 462 in its place. Tell students that this represents the number of girls in the entire school. It is not as easy to work with numbers this large using tables.

Link to Today's Concept

Tell students that in today's lesson, we are going to discuss another model for visualizing proportions. This tool is called a **tape diagram**. This tool helps us solve proportions with greater values like the one we just looked at.

Demonstrate

Engagement Strategy: Teacher Modeling

Demonstrate tape diagrams in the following way:

- Have students look at **Example 1** on page 178 of the *Student Text*. Tell students we will now learn how to work with tape diagrams to solve proportion problems. Have a volunteer read the problem aloud. Be sure students understand all the parts of the proportion as they look at the table. What is the ratio that is known? ($\frac{2}{3}$) What else do we know? (*total number of chocolate cupcakes = 342*) What number are we trying to find? (*total number of red velvet cupcakes*)
- Ask students to try the vertical strategy for solving the problem. Even though 342 is a multiple of 3, the number that 3 is multiplied by to get 342 is not simple to compute just by looking at it like the other problems we have worked with.
- Have students look at the tape diagram at the bottom of the page. A tape diagram is made of small rectangles that we draw to represent the simple ratio in our problem. The given ratio of red velvet cupcakes to chocolate cupcakes is 2 to 3. Point out that we have drawn two rectangles for the red velvet cupcakes and three rectangles for the chocolate cupcakes. Explain that each of these rectangles represents a fair share amount, which means that the same number is written inside each rectangle.

Lesson 10 | Problem Solving: ▶Introduction to Tape Diagrams
Monitoring Progress: ▶Quiz 2

▶Problem Solving: Introduction to Tape Diagrams

Vocabulary
tape diagram

How do tape diagrams help us solve proportion problems?

We have used tables to solve ratio and proportion problems. Tables work well with small numbers and when the multiplication is relatively easy. All of this changes when we work with large numbers in word problems. **Tape diagrams** help us make sense of these problems.

Example 1

Phillips Bakery makes two kinds of cupcakes, a chocolate cupcake and a red velvet cupcake. For every 2 red velvet cupcakes, the bakery makes 3 chocolate cupcakes. The bakery got a big order this morning for both kinds of cupcakes, so they made 342 chocolate cupcakes. How many red velvet cupcakes did they make?

Let's use the variable y to represent the number of red velvet cupcakes. We see the difficulty trying to figure out the multiplication when we put the numbers in a table.

Number of red velvet cupcakes	2	y
Number of chocolate cupcakes	3	342

Tape diagrams give us another way to solve this problem. We will use rectangles to represent the ratio of red velvet cupcakes to chocolate cupcakes.

The next step is how we think about each of the rectangles. They are fair shares. That means the same number will be written in each rectangle.

178 Unit 2 • Lesson 10

254 Unit 2 • Lesson 10

TransMath uses well-chosen representations throughout the curriculum to help students understand difficult concepts and think precisely about mathematical topics. This example (from page 254 of the Teacher's Guide, Volume 1, Level 3) introduces tape diagrams by drawing on a previous representation for thinking about ratios and proportions: A table.

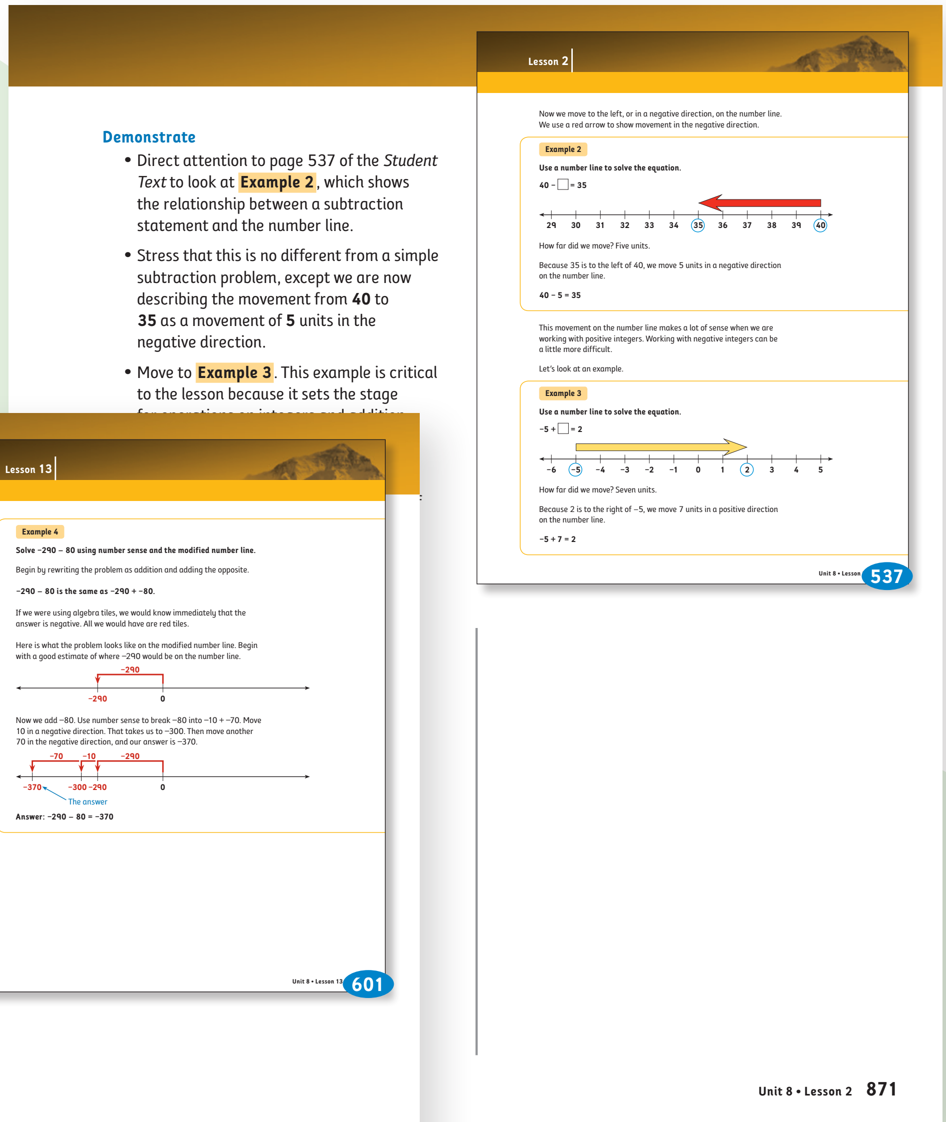
RECOMMENDATION 4: NUMBER LINES

Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

TransMath encourages the use of number lines in two important topical areas in lessons: fractions/decimals and integers. This approach aligns with the IES Guide, which recommends number lines to emphasize the magnitude of numbers and helps students understand "the concepts underlying the operations."

Examples

This example shows the use of two kinds of number lines when getting students to think about operations on integers. The first example (from page 871 of the Teacher's Guide, Volume 2, Level 2) shows movement on a number line.



Demonstrate

- Direct attention to page 537 of the *Student Text* to look at **Example 2**, which shows the relationship between a subtraction statement and the number line.
- Stress that this is no different from a simple subtraction problem, except we are now describing the movement from **40** to **35** as a movement of **5** units in the negative direction.
- Move to **Example 3**. This example is critical to the lesson because it sets the stage

Lesson 2

Now we move to the left, or in a negative direction, on the number line. We use a red arrow to show movement in the negative direction.

Example 2

Use a number line to solve the equation.

40 – ☐ = 35

How far did we move? Five units.

Because 35 is to the left of 40, we move 5 units in a negative direction on the number line.

40 – 5 = 35

This movement on the number line makes a lot of sense when we are working with positive integers. Working with negative integers can be a little more difficult.

Let's look at an example.

Example 3

Use a number line to solve the equation.

–5 + ☐ = 2

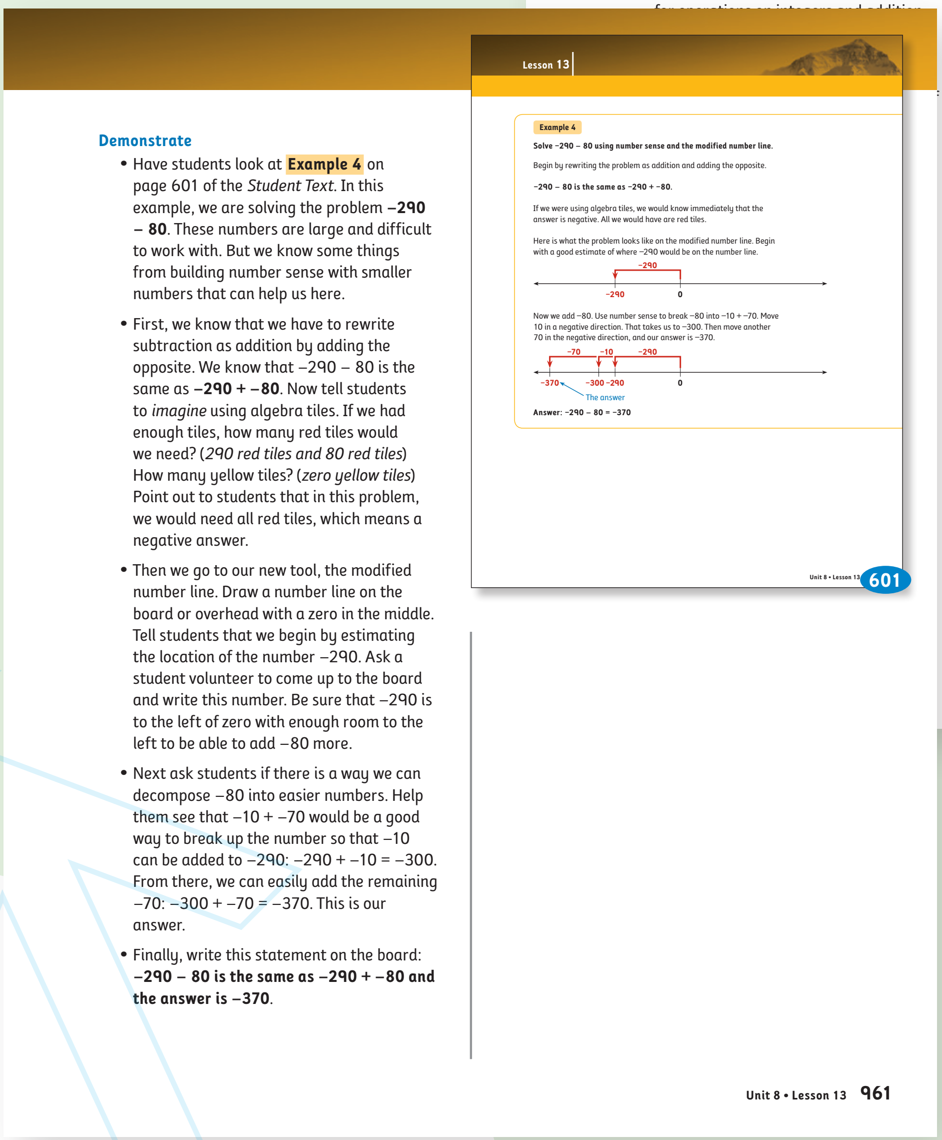
How far did we move? Seven units.

Because 2 is to the right of –5, we move 7 units in a positive direction on the number line.

–5 + 7 = 2

Unit 8 • Lesson 537

Unit 8 • Lesson 2 871



Demonstrate

- Have students look at **Example 4** on page 601 of the *Student Text*. In this example, we are solving the problem **–290 – 80**. These numbers are large and difficult to work with. But we know some things from building number sense with smaller numbers that can help us here.
- First, we know that we have to rewrite subtraction as addition by adding the opposite. We know that –290 – 80 is the same as **–290 + –80**. Now tell students to *imagine* using algebra tiles. If we had enough tiles, how many red tiles would we need? (*290 red tiles and 80 red tiles*) How many yellow tiles? (*zero yellow tiles*) Point out to students that in this problem, we would need all red tiles, which means a negative answer.
- Then we go to our new tool, the modified number line. Draw a number line on the board or overhead with a zero in the middle. Tell students that we begin by estimating the location of the number –290. Ask a student volunteer to come up to the board and write this number. Be sure that –290 is to the left of zero with enough room to the left to be able to add –80 more.
- Next ask students if there is a way we can decompose –80 into easier numbers. Help them see that –10 + –70 would be a good way to break up the number so that –10 can be added to –290: –290 + –10 = –300. From there, we can easily add the remaining –70: –300 + –70 = –370. This is our answer.
- Finally, write this statement on the board: **–290 – 80 is the same as –290 + –80 and the answer is –370.**

Lesson 13

Example 4

Solve –290 – 80 using number sense and the modified number line.

Begin by rewriting the problem as addition and adding the opposite.

–290 – 80 is the same as –290 + –80.

If we were using algebra tiles, we would know immediately that the answer is negative. All we would have are red tiles.

Here is what the problem looks like on the modified number line. Begin with a good estimate of where –290 would be on the number line.

Now we add –80. Use number sense to break –80 into –10 + –70. Move 10 in a negative direction. That takes us to –300. Then move another 70 in the negative direction, and our answer is –370.

Answer: –290 – 80 = –370

Unit 8 • Lesson 13 601

Unit 8 • Lesson 13 961

This example (from page 961 of the Teacher's Guide, Volume 2, Level 2) illustrates how we think about operations on large numbers using a different kind of number: An open number line.

RECOMMENDATION 5: WORD PROBLEMS

Provide deliberate instruction about word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

A key feature of *TransMath* is the significant portion of each day's lesson dedicated to problem solving. This approach helps students apply and expand their understanding while fostering classroom discussions. In alignment with recommendations, *TransMath* emphasizes "types of word problems" where relevant.

FOR EXAMPLE: Level 1 focuses on rate problems to build multiplicative thinking and differentiates between part-whole and part-part-whole word problems. Level 3 explores ratios, proportions, and algebra problem-solving, linking word problems to tables, graphs, and equations.

TransMath also aligns with Common Core State Standards by addressing "everyday/real-life" problems relevant to students' experiences.

LEVEL 1: Features problems involving whole-number operations in data-related contexts.

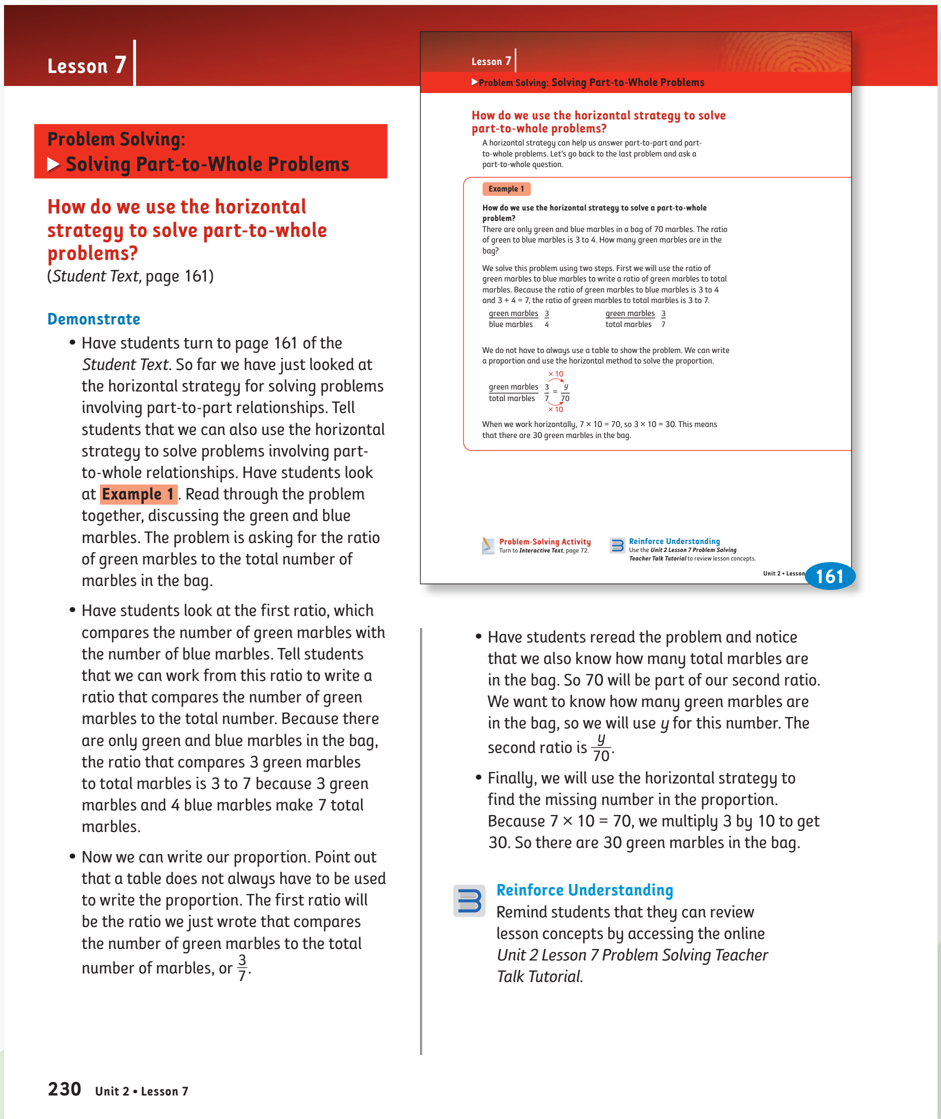
LEVEL 2: Contextualizes fractions through scenarios like home repair and food preparation.

LEVEL 3: Includes ratio, proportion, and algebra problems tied to real-life situations.

The program often revisits the same context across multiple lessons, allowing students to deepen their "mental model" of problem-solving scenarios and see how mathematical concepts apply in different ways.

Additionally, *TransMath* pays close attention to the structure of word problems. For instance, in a food-preparation scenario, students may calculate total dry ingredients for a dessert using fractions (e.g., 4 ¼ cups of flour and ½ cup of sugar). Such part-part-whole problems connect math with practical, relatable tasks.

Examples



Lesson 7

Problem Solving:
▶ Solving Part-to-Whole Problems

How do we use the horizontal strategy to solve part-to-whole problems?
(*Student Text*, page 161)

Demonstrate

- Have students turn to page 161 of the *Student Text*. So far we have just looked at the horizontal strategy for solving problems involving part-to-part relationships. Tell students that we can also use the horizontal strategy to solve problems involving part-to-whole relationships. Have students look at **Example 1**. Read through the problem together, discussing the green and blue marbles. The problem is asking for the ratio of green marbles to the total number of marbles in the bag.
- Have students look at the first ratio, which compares the number of green marbles with the number of blue marbles. Tell students that we can work from this ratio to write a ratio that compares the number of green marbles to the total number. Because there are only green and blue marbles in the bag, the ratio that compares 3 green marbles to total marbles is 3 to 7 because 3 green marbles and 4 blue marbles make 7 total marbles.
- Now we can write our proportion. Point out that a table does not always have to be used to write the proportion. The first ratio will be the ratio we just wrote that compares the number of green marbles to the total number of marbles, or $\frac{3}{7}$.

Lesson 7

▶Problem Solving: Solving Part-to-Whole Problems

How do we use the horizontal strategy to solve part-to-whole problems?

A horizontal strategy can help us answer part-to-part and part-to-whole problems. Let's go back to the last problem and ask a part-to-whole question.

Example 1

How do we use the horizontal strategy to solve a part-to-whole problem?
There are only green and blue marbles in a bag of 70 marbles. The ratio of green to blue marbles is 3 to 4. How many green marbles are in the bag?

We solve this problem using two steps. First we will use the ratio of green marbles to blue marbles to write a ratio of green marbles to total marbles. Because the ratio of green marbles to blue marbles is 3 to 4 and 3 + 4 = 7, the ratio of green marbles to total marbles is 3 to 7.

$\frac{\text{green marbles}}{\text{blue marbles}} \quad \frac{3}{4}$ $\qquad$ $\frac{\text{green marbles}}{\text{total marbles}} \quad \frac{3}{7}$

We do not have to always use a table to show the problem. We can write a proportion and use the horizontal method to solve the proportion.

$\frac{\text{green marbles}}{\text{total marbles}} \quad \frac{3}{7} = \frac{y}{70}$ (× 10, × 10)

When we work horizontally, 7 × 10 = 70, so 3 × 10 = 30. This means that there are 30 green marbles in the bag.

Problem-Solving Activity Turn to ***Interactive Text***, page 72.

Reinforce Understanding Use the ***Unit 2 Lesson 7 Problem Solving Teacher Talk Tutorial*** to review lesson concepts.

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- Have students reread the problem and notice that we also know how many total marbles are in the bag. So 70 will be part of our second ratio. We want to know how many green marbles are in the bag, so we will use y for this number. The second ratio is $\frac{y}{70}$.
- Finally, we will use the horizontal strategy to find the missing number in the proportion. Because 7 × 10 = 70, we multiply 3 by 10 to get 30. So there are 30 green marbles in the bag.

Reinforce Understanding
Remind students that they can review lesson concepts by accessing the online *Unit 2 Lesson 7 Problem Solving Teacher Talk Tutorial.*

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Types of word problems can be more challenging at a higher level. This example (from page 230 of the Teacher's Guide, Volume 1, Level 3) illustrates how TransMath helps students distinguish between part-to-whole problems and part-part-whole problems. This distinction is key to helping students think about ratios.

RECOMMENDATION 6: TIMED ACTIVITIES

Regularly include timed activities as one way to build fluency in mathematics.

TransMath encourages the systematic development of math facts across Level 1 of the program, and each lesson of the program across all three levels begins with a 5-minute warm up during which students review basic number skills (math facts, simple operations on fractions, operations on integers). This reinforces previous learning and provides a brief review of numbers and operation on numbers that will be used in the day's lesson.

Each day, *TransMath* begins with a brief warm-up activity found in the student workbook. The exercises are brief, self-explanatory, and are designed around two purposes:

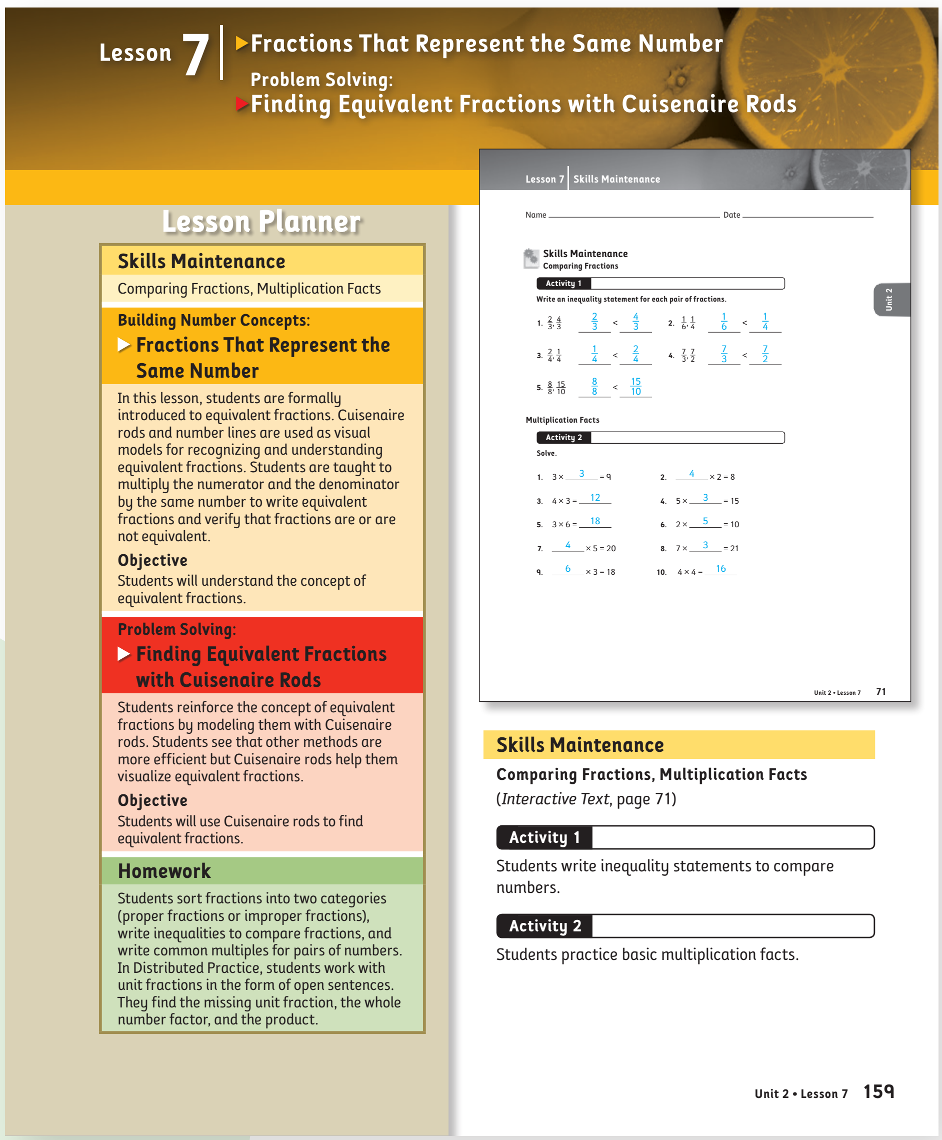
1) Preview numbers that will be used in the lesson

2) Provide distributed practice on key skills

There is a difference between automaticity and fluency in mathematics. *TransMath* encourages both kinds of thinking. Automaticity is usually associated with math facts, and *TransMath* stresses this skill throughout the first two levels of the program. Fluency, on the other hand, is being able to think about a problem without an extended amount of time, and it also means being able to use an algorithm like multiplication to find an answer without stopping and starting. (This example is taken from page 159 of the Teacher's Guide, Volume 1, Level 2.)

Activity 1 shown here is an example of fluency, and students have already worked a great deal on strategies for comparing the size or magnitude of fractions. Activity 2 is automaticity practice on multiplication facts. This is highly relevant, because students are going to begin computing equivalent fractions and in doing so, they need to find common denominators.

Examples



Lesson 7 | Fractions That Represent the Same Number

Problem Solving: Finding Equivalent Fractions with Cuisenaire Rods

Lesson Planner

Skills Maintenance

Comparing Fractions, Multiplication Facts

Building Number Concepts:

Fractions That Represent the Same Number

In this lesson, students are formally introduced to equivalent fractions. Cuisenaire rods and number lines are used as visual models for recognizing and understanding equivalent fractions. Students are taught to multiply the numerator and the denominator by the same number to write equivalent fractions and verify that fractions are or are not equivalent.

Objective

Students will understand the concept of equivalent fractions.

Problem Solving:

Finding Equivalent Fractions with Cuisenaire Rods

Students reinforce the concept of equivalent fractions by modeling them with Cuisenaire rods. Students see that other methods are more efficient but Cuisenaire rods help them visualize equivalent fractions.

Objective

Students will use Cuisenaire rods to find equivalent fractions.

Homework

Students sort fractions into two categories (proper fractions or improper fractions), write inequalities to compare fractions, and write common multiples for pairs of numbers. In Distributed Practice, students work with unit fractions in the form of open sentences. They find the missing unit fraction, the whole number factor, and the product.

Lesson 7 | Skills Maintenance

Name ______ Date ______

Skills Maintenance

Comparing Fractions

Activity 1

Write an inequality statement for each pair of fractions.

1. $\frac{2}{3}, \frac{4}{3}$ $\frac{2}{3} < \frac{4}{3}$
2. $\frac{1}{6}, \frac{1}{4}$ $\frac{1}{6} < \frac{1}{4}$
3. $\frac{2}{4}, \frac{1}{4}$ $\frac{1}{4} < \frac{2}{4}$
4. $\frac{7}{3}, \frac{7}{2}$ $\frac{7}{3} < \frac{7}{2}$
5. $\frac{8}{8}, \frac{15}{10}$ $\frac{8}{8} < \frac{15}{10}$

Multiplication Facts

Activity 2

Solve.

1. $3 \times$ 3 $= 9$
2. 4 $\times 2 = 8$
3. $4 \times 3 =$ 12
4. $5 \times$ 3 $= 15$
5. $3 \times 6 =$ 18
6. $2 \times$ 5 $= 10$
7. 4 $\times 5 = 20$
8. $7 \times$ 3 $= 21$
9. 6 $\times 3 = 18$
10. $4 \times 4 =$ 16

Unit 2

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Skills Maintenance

Comparing Fractions, Multiplication Facts

(*Interactive Text*, page 71)

Activity 1

Students write inequality statements to compare numbers.

Activity 2

Students practice basic multiplication facts.

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